B.Sc Part I

Areas of plane curves

Area of a curve given by Cartesian equation

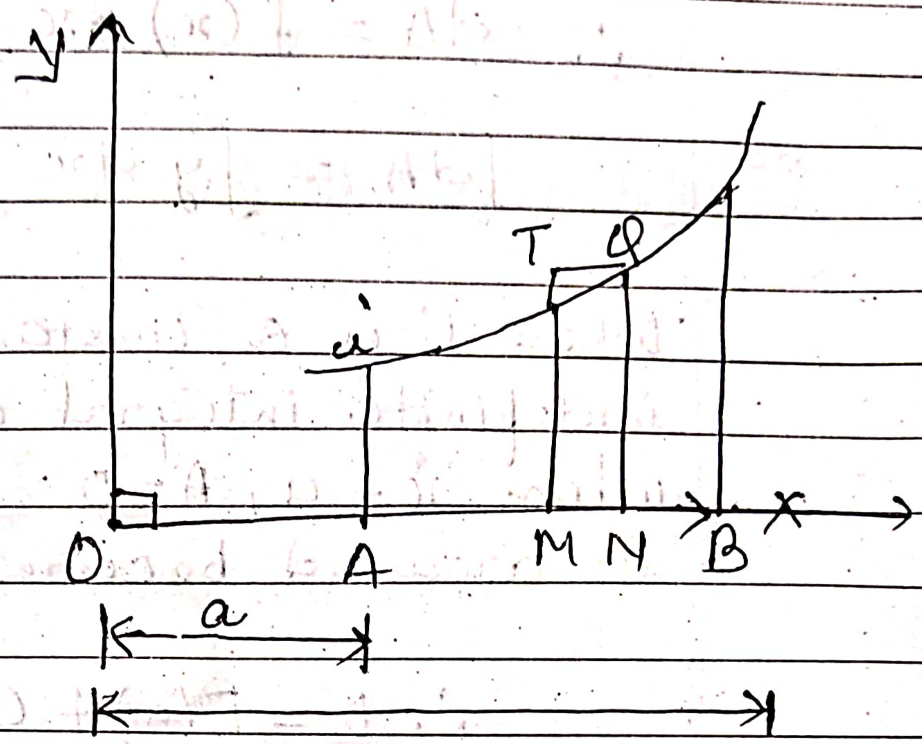
Referred to rectangular axes of co-ordinates, the area included between the curve, the axis of x and two ordinates $x = a$ and $x = b$ is represented by the definite integral.

$$\text{Area} = \int_a^b y\,dx.$$

where $y = f(x)$ is equation of the curve.

Proof Let CD be the curve whose equation is $y = f(x)$, CA and DB are the two ordinates at $x = a$ and $x = b$ respectively $b > a$

For convenience we suppose that the ordinates are increasing between the given interval (a, b), the curve is also continuous and finite in interval. It is required to find the area ABCD



Let the co-ordinates of P be (x, y) and of Q, $(x+\delta x, y+\delta y)$, so that $OM = x$, $ON = x+\delta x$, $MP = y$ and $NQ = y+\delta y$

Let the Area AMPC be denoted by A. The increment δA in A corresponding to the change δx in x is the area PMNQP which lies between the area PMNQTP and PMNR i.e. δA lies between $(y+\delta y)\delta x$ and $y\delta x$.

Or, $(y+\delta y)\delta x > \delta A > y\delta x$

Or, $(y+\delta y) > \dfrac{\delta A}{\delta x} > y$

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$$\therefore \frac{dA}{dx} = \lim_{\delta x \to 0} \frac{\delta A}{\delta x} = y = f(x)$$

$$\therefore dA = f(x)\,dx = y\,dx$$

$$\int dA = \int y\,dx = F(x) + C.$$

Where C is a constant and F(x) is an indefinite integral of f(x). Now when $x = a$, $A = 0$. Also when $x = b$ the area A becomes required area A_1.

$$\therefore 0 = F(a) + C$$

$$A_1 = F(b) + C.$$

$$\therefore A_1 = F(b) - F(a) = \int_a^b f(x)\,dx.$$

$\therefore$ The definite $\int_a^b y\,dx$ represents the area bounded by the curve

$y = f(x)$, the x-Axis and the two fixed ordinates $x = a$ and $x = b$.